Complete Recovery of Correlations in an X-ray Beam

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Why do we recover wavefields

- Recover as much information about the wavefield as possible
- Compensate for the experimental conditions
- Relate measurement to structure
Seeing Phase

Refraction of light passing through water is a phase effect. The twinkling of a star is an analogous phenomenon.
One approach to solution

Assume the paraxial approximation:

\[
\frac{\partial I(\vec{r}_\perp)}{\partial z} = \frac{\lambda}{2\pi} \nabla_\perp \cdot [I(\vec{r}_\perp) \nabla_\perp \phi(\vec{r}_\perp)]
\]

This equation has a unique solution in the case where the phase front is not discontinuous.

A measurement of the probability (intensity) and its longitudinal derivative specifies the complete wave (function) over all space!

X-Ray Complex Phase Tomography
Change in measured intensity is formally identical to the Tol equation!

\[ \frac{1}{2} k_0 R \delta I_f (\vec{k}) = \nabla \cdot \left\{ I_f (\vec{k}) \nabla \Phi (\vec{k}) \right\} \]

Vortices are ubiquitous in the far-field and so this equation cannot be solved uniquely, except under very special conditions.
X-ray Vortex from Simple Three-Molecule Diffraction
Far-Field Diffraction with Cylindrical Incident Beam

Now consider cylindrical curvature incident. An identical argument gives:

\[ \frac{1}{2} k_0 R \Delta I_f (\vec{k}) \approx \frac{\partial}{\partial k_x} \left\{ I_f (\vec{k}) \frac{\partial \Phi (\vec{k})}{\partial k_x} \right\} \]

This may be directly integrated to obtain:

\[ I_f (\vec{k}) \frac{\partial \Phi (\vec{k})}{\partial k_x} = \frac{1}{2k_0} R \int \Delta I_f (\vec{k}) dk_x + g_x (k_y) \]
What Characterizes Wavefields?

For Gaussian statistics, a wavefield is fully characterised by the mutual coherence function.

This is a complex four-dimensional function describing the phase and visibility of fringes created by Young’s experiments as a function of the two-dimensional positions of the pinholes.

\[ J(\vec{r}_1, \vec{r}_2) \equiv \langle E(\vec{r}_1)E^*(\vec{r}_2) \rangle \]
What Characterizes Wavefields?

\[ J(x_1, x_2) \equiv \langle E(x_1)E^*(x_2) \rangle \]
The coherence function

We have recently measured the correlations for 7.9keV x-rays from the 2-ID-D beamline at the APS

We use the Wigner, or generalised radiance, function:

\[ B(x, u) \equiv \int J\left(x + \frac{q}{2}, x + \frac{q}{2}\right) \exp\left[-2\pi iqu / \lambda\right] dq \]
The Wigner Function

\[ u = \tan(\theta) \]

\[ B(x, \theta, z) = \int B(x, u) du \]

\[ x - zu \]
Wave propagation as geometry

The Wigner function has very nice properties in the paraxial approximation:

\[ B(x, u, z) = B(x - zu, u, 0) \]  \hspace{1cm} \text{Geometric Propagation}

\[ I(x, z) = \int B(x, u, z) du \]  \hspace{1cm} \text{Intensity as a sum over angles}
The Generalised Radiance Function
Phase Space Tomography

Contact Image –– No angular information

Fraunhofer diffraction –– only angular information

\[ \theta = \arctan \left( \frac{z}{k} \right) \]
Intensity distribution
Recovered Coherence Function
The Complex Degree of Coherence

We use the Wigner, or generalised radiance, function:

\[ \gamma(x_1, x_2) \equiv \frac{J(x_1, x_2)}{\sqrt{I(x_1)I(x_2)}} \]
Recovered Coherence Function
Is the field statistically stationary?

\[ X = X_1 - X_2 \]
Assembled data
Summary

• Can view phase as a rather geometric property of light.
• This yields methods that are quantitative.
• Can be applied to far-field diffraction
• Have developed techniques to obtain complex coherence function
• Results are in good agreement with expectations